Decision Making Under Severe Uncertainty: Solutions to Exercises

A Very Simple Example

Example from lecture:

	increase	*stay*	*p0*	*p1*	*lpr*	*upr*
increase			0.5	0.8		
stay			0.5	0.2		
machinery	440	260	350	404	350	404
overtime	420	300	360	396	360	396
nothing	370	370	370	370	370	370
overtime - machinery			+	-	-	
nothing - machinery			+	-	-	
machinery - overtime			-	+	-	
nothing - overtime			+	-	-	
machinery - nothing			-	+	-	
overtime - nothing			-	+	-	

	machinery	*overtime*	*nothing*
- machinery	**0**	-	-
- overtime	-	**0**	-
- nothing	-	-	**0**

- Gamma-maximin: {nothing}
- Gamma-maximax: {machinery}
- interval dominance: {machinery, overtime, nothing}
- maximality: {machinery, overtime, nothing}

After market research:

	increase	*stay*	*p0*	*p1*	*lpr*	*upr*
increase			0.6	0.65		
stay			0.4	0.35		
machinery	440	260	368	377	368	377
overtime	420	300	372	378	372	378
nothing	370	370	370	370	370	370
overtime - machinery			+	+	+	
nothing - machinery			+	-	-	
machinery - overtime			-	-	-	
nothing - overtime			-	-	-	

machinery - nothing		-	+	-	
overtime - nothing		+	+	+	

	machinery	*overtime*	*nothing*
- machinery	**0**	+	-
- overtime	-	**0**	-
- nothing	-	+	**0**

- Gamma-maximin: {overtime}
- Gamma-maximax: {overtime}
- interval dominance: {machinery, overtime}
- maximality: {overtime}

Saving Zion

	A	*B*	*C*	*p0*	*p1*	*p2*	*lpr*	*upr*
A				0.1	0.4	0.3		
B				0.45	0.3	0.2		
C				0.45	0.3	0.5		
left	-10	-5	10	1.25	-2.5	1	-2.5	1.25
right	1	1	1	1	1	1	1	1
right - left				-	+	**0**	-	
left - right				+	-	**0**	-	

	left	*right*
- left	**0**	-
- right	-	**0**

- Gamma-maximin: {right}
- Gamma-maximax: {left}
- interval dominance: {left, right}
- maximality: {left, right}

Risky Investment

	improve	*remain*	*worsen*	*p0*	*p1*	*lpr*	*upr*
improve				0	0.3		
remain				0.6	0.3		
worsen				0.4	0.4		
d1	100	50	-25	20	35	20	35
d2	75	50	0	30	37.5	30	37.5

d3	60	55	10	37	38.5	37	38.5
d4	35	35	35	35	35	35	35
d2 - d1				+	+	+	
d3 - d1				+	+	+	
d4 - d1				+	**0**	**0**	
d1 - d2				-	-	-	
d3 - d2				+	+	+	
d4 - d2				+	-	-	
d1 - d3				-	-	-	
d2 - d3				-	-	-	
d4 - d3				-	-	-	
d1 - d4				-	**0**	-	
d2 - d4				-	+	-	
d3 - d4				+	+	+	

	d1	*d2*	*d3*	*d4*
- d1	**0**	+	+	**0**
- d2	-	**0**	+	-
- d3	-	-	**0**	-
- d4	-	-	+	**0**

- Gamma-maximin: {3}
- Gamma-maximax: {3}
- interval dominance: {2, 3}
- maximality: {3}

Solution to Exercise 5:

- c<37
    - Gamma-maximin: {3}
    - Gamma-maximax: {3}
    - interval dominance: {2, 3}
    - maximality: {3}
- c=37
    - Gamma-maximin: {3, 4}
    - Gamma-maximax: {3}
    - interval dominance: {2, 3, 4}
    - maximality: {3, 4}
- 37<c<37.5
    - Gamma-maximin: {4}

- Gamma-maximax: {3}
- interval dominance: {2, 3, 4}
- maximality: {3, 4}

- 37.5<c<38.5
  - Gamma-maximin: {4}
  - Gamma-maximax: {3}
  - interval dominance: {3, 4}
  - maximality: {3, 4}
- 38.5<c
  - Gamma-maximin: {4}
  - Gamma-maximax: {4}
  - interval dominance: {4}
  - maximality: {4}

Oil Wildcatter

Normal Form Backward Induction

T1 Branch

	S1	*S2*	*S3*	*p0*	*p1*	*p2*	*lpr*	*upr*
S1				0.531	0.667	0.817		
S2				0.408	0.222	0.122		
S3				0.061	0.111	0.061		
d1	-1	-1	-1	-1	-1	-1	-1	-1
d2	-8	4	19	-1.457	-2.339	-4.889	-4.889	-1.457
d2 - d1				-	-	-	-	
d1 - d2				+	+	+	+	

	d1	*d2*
- d1	**0**	-
- d2	+	**0**

So {d1} is maximal in the T1 branch.

T2 Branch

	S1	*S2*	*S3*	*p0*	*p1*	*p2*	*lpr*	*upr*
S1				0.065	0.125	0.5		
S2				0.87	0.75	0.5		
S3				0.065	0.125	0		

d1	-1	-1	-1	-1	-1	-1	-1	-1
d2	-8	4	19	4.195	4.375	-2	-2	4.375
d2 - d1				+	+	-	-	
d1 - d2				-	-	+	-	

	d1	*d2*
- d1	**0**	-
- d2	-	**0**

So {d1,d2} is maximal in the T2 branch.

T3 Branch

	S1	*S2*	*S3*	*p0*	*p1*	*p2*	*lpr*	*upr*
S1				0.061	0.111	0.061		
S2				0.408	0.222	0.122		
S3				0.531	0.667	0.817		
d1	-1	-1	-1	-1	-1	-1	-1	-1
d2	-8	4	19	11.233	12.673	15.523	11.233	15.523
d2 - d1				+	+	+	+	
d1 - d2				-	-	-	-	

	d1	*d2*
- d1	**0**	+
- d2	-	**0**

So {d2} is maximal in the T3 branch.

dTc branch

	T1S1	*T1S2*	*T1S3*	*T2S1*	*T2S2*	*T2S3*	*T3S1*	*T3S2*	*T3S3*	*p0*	*p1*	*p2*	*p3*	*p4*	*p5*	*lpr*	*upr*
T1S1										0.18	0.18	0.18	0.18	0.4	0.26		
T1S2										0.06	0.06	0.06	0.06	0.06	0.2		
T1S3										0.03	0.03	0.03	0.03	0.03	0.03		
T2S1										0.03	0.03	0.03	0.23	0.03	0.03		
T2S2										0.18	0.18	0.4	0.23	0.18	0.18		
T2S3										0.03	0.03	0.03	0	0.03	0.03		
T3S1										0.03	0.03	0.03	0.03	0.03	0.03		
T3S2										0.06	0.2	0.06	0.06	0.06	0.06		
T3S3										0.4	0.26	0.18	0.18	0.18	0.18		
d1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
d2	-7	5	20	-7	5	20	-7	5	20	9.02	6.92	5.72	2.87	3.08	4.76	2.87	9.02
d2 - d1										+	+	+	+	+	+	+	

d1 - d2		-	-	-	-	-	-	-	

	d1	*d2*
- d1	**0**	+
- d2	-	**0**

So {d2} is maximal in the dTc branch.

dT Branch

	T1S1	*T1S2*	*T1S3*	*T2S1*	*T2S2*	*T2S3*	*T3S1*	*T3S2*	*T3S3*	*p0*	*p1*	*p2*	*p3*	*p4*	*p5*	*lpr*	*upr*
T1S1										0.18	0.18	0.18	0.18	0.4	0.26		
T1S2										0.06	0.06	0.06	0.06	0.06	0.2		
T1S3										0.03	0.03	0.03	0.03	0.03	0.03		
T2S1										0.03	0.03	0.03	0.23	0.03	0.03		
T2S2										0.18	0.18	0.4	0.23	0.18	0.18		
T2S3										0.03	0.03	0.03	0	0.03	0.03		
T3S1										0.03	0.03	0.03	0.03	0.03	0.03		
T3S2										0.06	0.2	0.06	0.06	0.06	0.06		
T3S3										0.4	0.26	0.18	0.18	0.18	0.18		
s1	-1	-1	-1	-1	-1	-1	-8	4	19	7.09	4.99	2.69	2.69	2.69	2.69	2.69	7.09
s2	-1	-1	-1	-8	4	19	-8	4	19	8.38	6.28	5.08	2.23	3.98	3.98	2.23	8.38
s2 - s1										+	+	+	-	+	+	-	
s1 - s2										-	-	-	+	-	-	-	

	s1	*s2*
- s1	**0**	-
- s2	-	**0**

So both strategies {s1,s2} are maximal in the dT branch.

Root node:

	T1S1	*T1S2*	*T1S3*	*T2S1*	*T2S2*	*T2S3*	*T3S1*	*T3S2*	*T3S3*	*p0*	*p1*	*p2*	*p3*	*p4*	*p5*	*lpr*	*upr*
T1S1										0.18	0.18	0.18	0.18	0.4	0.26		
T1S2										0.06	0.06	0.06	0.06	0.06	0.2		
T1S3										0.03	0.03	0.03	0.03	0.03	0.03		
T2S1										0.03	0.03	0.03	0.23	0.03	0.03		
T2S2										0.18	0.18	0.4	0.23	0.18	0.18		
T2S3										0.03	0.03	0.03	0	0.03	0.03		
T3S1										0.03	0.03	0.03	0.03	0.03	0.03		
T3S2										0.06	0.2	0.06	0.06	0.06	0.06		
T3S3										0.4	0.26	0.18	0.18	0.18	0.18		

s1	-1	-1	-1	-1	-1	-1	-8	4	19	7.09	4.99	2.69	2.69	2.69	2.69	2.69	7.09
s2	-1	-1	-1	-8	4	19	-8	4	19	8.38	6.28	5.08	2.23	3.98	3.98	2.23	8.38
d2	-7	5	20	-7	5	20	-7	5	20	9.02	6.92	5.72	2.87	3.08	4.76	2.87	9.02
s2 - s1										+	+	+	-	+	+	-	
d2 - s1										+	+	+	+	+	+	+	
s1 - s2										-	-	-	+	-	-	-	
d2 - s2										+	+	+	+	-	+	-	
s1 - d2										-	-	-	-	-	-	-	
s2 - d2										-	-	-	-	+	-	-	

	s1	*s2*	*d2*
- s1	**0**	-	+
- s2	-	**0**	-
- d2	-	-	**0**

So {s2,d2} is maximal.

Toy Example

Note, for brevity, the tables below write D1 for delta1 and D2 for delta2.

Normal Form

	S1E1	*S1E2*	*S2*	*p0*	*p1*	*lpr*	*upr*
S1E1				0.2	0.1		
S1E2				0.3	0.4		
S2				0.5	0.5		
d1(S1 D1)	1	4	2	2.4	2.7	2.4	2.7
d1(S1 D2)	2.5	2.5	2	2.25	2.25	2.25	2.25
d2	2.3	2.3	2.3	2.3	2.3	2.3	2.3
d1(S1 D2) - d1(S1 D1)				-	-	-	
d2 - d1(S1 D1)				-	-	-	
d1(S1 D1) - d1(S1 D2)				+	+	+	
d2 - d1(S1 D2)				+	+	+	
d1(S1 D1) - d2				+	+	+	
d1(S1 D2) - d2				-	-	-	

	d1(S1 D1)	*d1(S1 D2)*	*d2*
- d1(S1 D1)	**0**	-	-
- d1(S1 D2)	+	**0**	+
- d2	+	-	**0**

So {d1(S1 delta1)} is maximal.

Normal Form Backward Induction

S1 Branch

	E1	*E2*	*p0*	*p1*	*lpr*	*upr*
E1			0.4	0.2		
E2			0.6	0.8		
D1	1	4	2.8	3.4	2.8	3.4
D2	2.5	2.5	2.5	2.5	2.5	2.5
D2 - D1			-	-	-	
D1 - D2			+	+	+	

	D1	*D2*
- D1	**0**	-
- D2	+	**0**

So {delta1} is maximal.

Root Node

	S1E1	*S1E2*	*S2*	*p0*	*p1*	*lpr*	*upr*
S1E1				0.2	0.1		
S1E2				0.3	0.4		
S2				0.5	0.5		
d1(S1 D1)	1	4	2	2.4	2.7	2.4	2.7
d2	2.3	2.3	2.3	2.3	2.3	2.3	2.3
d2 - d1(S1 D1)				-	-	-	
d1(S1 D1) - d2				+	+	+	

	d1(S1 D1)	*d2*
- d1(S1 D1)	**0**	-
- d2	+	**0**

So {d1(S1 delta1)} is maximal.

Extra Question

d2 becomes uniquely maximal if it is assigned a value strictly larger than 2.7.

Lake District

Normal Form Backward Induction

S1 Branch

	E1	*E2*	*p0*	*p1*	*p2*	*lpr*	*upr*
E1			0.591	0.7	0.747		
E2			0.409	0.3	0.253		
d1	9	14	11.045	10.5	10.265	10.265	11.045
d2	4	19	10.135	8.5	7.795	7.795	10.135
d2 - d1			-	-	-	-	
d1 - d2			+	+	+	+	

	d1	*d2*
- d1	**0**	-
- d2	+	**0**

So {d1} is maximal in the S1 branch.

S2 Branch

	E1	*E2*	*p0*	*p1*	*p2*	*lpr*	*upr*
E1			0.157	0.2	0.374		
E2			0.843	0.8	0.626		
d1	9	14	13.215	13	12.13	12.13	13.215
d2	4	19	16.645	16	13.39	13.39	16.645
d2 - d1			+	+	+	+	
d1 - d2			-	-	-	-	

	d1	*d2*
- d1	**0**	+
- d2	-	**0**

So {d2} is maximal in the S1 branch.

dSc Branch

	S1E1	*S1E2*	*S2E1*	*S2E2*	*p0*	*p1*	*p2*	*p3*	*lpr*	*upr*
S1E1					0.378	0.378	0.378	0.478		
S1E2					0.162	0.162	0.262	0.162		
S2E1					0.072	0.172	0.072	0.072		
S2E2					0.388	0.288	0.288	0.288		
d1	10	15	10	15	12.75	12.25	12.75	12.25	12.25	12.75
d2	5	20	5	20	13.25	11.75	13.25	11.75	11.75	13.25
d2 - d1					+	-	+	-	-	
d1 - d2					-	+	-	+	-	

	d1	*d2*

- d1	**0**	**-**
- d2	**-**	**0**

So both options {d1,d2} are maximal in the dSc branch.

dS Branch

The solution is trivial, as there is only a single gamble: 9, 14, 4, 19.

Root node

	S1E1	*S1E2*	*S2E1*	*S2E2*	*p0*	*p1*	*p2*	*p3*	*lpr*	*upr*
S1E1					0.378	0.378	0.378	0.478		
S1E2					0.162	0.162	0.262	0.162		
S2E1					0.072	0.172	0.072	0.072		
S2E2					0.388	0.288	0.288	0.288		
s1	9	14	4	19	13.33	11.83	12.83	12.33	11.83	13.33
d1	10	15	10	15	12.75	12.25	12.75	12.25	12.25	12.75
d2	5	20	5	20	13.25	11.75	13.25	11.75	11.75	13.25
d1 - s1					-	+	-	-	-	
d2 - s1					-	-	+	-	-	
s1 - d1					+	-	+	+	-	
d2 - d1					+	-	+	-	-	
s1 - d2					+	+	-	+	-	
d1 - d2					-	+	-	+	-	

	s1	*d1*	*d2*
- s1	**0**	**-**	**-**
- d1	**-**	**0**	**-**
- d2	**-**	**-**	**0**

So all three remaining normal form decisions {s1,d1,d2} are maximal.

Value of Information

A simple check reveals that s1 is no longer optimal as soon as c exceeds 1.58 (this would increase the lower prevision of d1 - s1 by 0.58, which is just enough to make it zero).