Solutions

1. (a) $P = \begin{bmatrix} 0.2 & 0.4 & 0.4 \\ 0.4 & 0.2 & 0.4 \\ 0.4 & 0.4 & 0.2 \end{bmatrix}$

   (b) $P(X_3 = x_3 | X_0 = x_1) = 0.336$

   (c) Yes, the limit distribution is $q^\infty = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$

   (d) $\tau(P) = 0.2$

   (e) $d(q^5, q^\infty) \leq d(q^0, q^\infty)\tau(P)^5 \leq 0.2^5$

2. (a) No, $\overline{P}' = \begin{bmatrix} 0.6 & 0.5 & 0.5 \\ 0.4 & 0.7 & 0.4 \\ 0.8 & 0.2 & 0.3 \end{bmatrix}$

   (b) $\underline{P}(X_2 = x_2 | X_0 = x_1) = 0.13$

   (c) $U = \begin{bmatrix} 0.6 & 0.5 & 0.7 & 0.8 & 0.9 & 0.8 \\ 0.4 & 0.7 & 0.8 & 0.4 & 0.6 & 0.9 \\ 0.8 & 0.2 & 0.9 & 0.3 & 0.9 & 0.4 \end{bmatrix}$

   (d) $\tau = U_3(\{x_3\}) - L_2(\{x_1\}) = 0.8 - 0.1 = 0.7$
   $\rho = U(\{x_2\}) - U(\{x_3\}) = 0.5$

3. (a) $\underline{P} = \begin{bmatrix} 0.2 & 0.3 & 0.3 \\ 0.3 & 0.2 & 0.3 \\ 0.3 & 0.3 & 0.2 \end{bmatrix}$ and $\overline{P} = \begin{bmatrix} 0.2 & 0.5 & 0.5 \\ 0.5 & 0.2 & 0.5 \\ 0.5 & 0.5 & 0.2 \end{bmatrix}$

   (b) $\underline{P} = \begin{bmatrix} 0 & 0.1 & 0.1 \\ 0.1 & 0 & 0.1 \\ 0.1 & 0.1 & 0 \end{bmatrix}$ and $\overline{P} = \begin{bmatrix} 0.6 & 0.9 & 0.9 \\ 0.9 & 0.6 & 0.9 \\ 0.9 & 0.9 & 0.4 \end{bmatrix}$

   (c) 0.28 / 0.04

4. (a) $\underline{P} = \begin{bmatrix} 0.4 & 0 & 0.2 \\ 0 & 0.5 & 0.3 \\ 0 & 0 & 0.5 \end{bmatrix}$ and $\overline{P} = \begin{bmatrix} 0.8 & 0.4 & 0.4 \\ 0.2 & 0.7 & 0.5 \\ 0.1 & 0.5 & 1 \end{bmatrix}$

   (b) 0.29