Inference & Desirability: Exercise solutions*

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1 Reasoning about and with sets of desirable gambles

1. Possibility space $\{a, b\}$; given are assessments

$$\begin{aligned}
\mathcal{A}_1 &\coloneqq \{(-1000, 1)\},\\
\mathcal{A}_2 &\coloneqq \{(-1000, 0), (\tfrac{1}{4}, \tfrac{1}{2}), (6, 3)\},\\
\mathcal{A}_3 &\coloneqq \{(-1000, 1), (\tfrac{1}{4}, -\tfrac{1}{2})\},\\
\mathcal{A}_4 &\coloneqq \{(-1, 2), (\tfrac{1}{2}, -\tfrac{1}{4})\}.
\end{aligned}$$

   1.1 Does $\mathcal{A}_i$ avoid sure loss?

   1.2 Does $\mathcal{A}_i$ avoid partial loss?

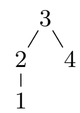
   1.3 Does $\operatorname{posi}(\mathcal{A}_i)$ accept sure gain?

   1.4 Does $\operatorname{posi}(\mathcal{A}_i)$ accept partial gain?

   1.5 If $\mathcal{A}_i$ avoids sure loss, describe $\mathcal{E}(\mathcal{A}_i)$ by giving its extreme rays (as sup-norm one vectors).

   1.6 Order all of the resulting $\mathcal{E}(\mathcal{A}_i)$ according to how committal they are.

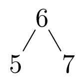
i	$\mathcal{A}_i$	asl	apl	asg	apg	$\mathcal{E}(\mathcal{A}_i)$	coh
1	$\{(-1, \frac{1}{1000})\}$	y	y	n	n	$\operatorname{posi}(\{(-1, \frac{1}{1000}), I_a\})$	y
2	$\{(-1, 0), (\frac{1}{2}, 1), (1, \frac{1}{2})\}$	y	n	n	n	$\operatorname{posi}(\{(-1, 0), I_a\})$	n
3	$\{(-1, \frac{1}{1000}), (\frac{1}{2}, -1)\}$	n	n	n	n	$\mathcal{L}(\{a, b\})$	n
4	$\{(-\frac{1}{2}, 1), (1, -\frac{1}{2})\}$	y	y	y	y	$\operatorname{posi}(\mathcal{A}_4)$	y



2. Possibility space $\{a, b, c\}$; given are assessments

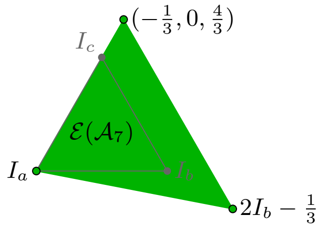
$$\begin{aligned}
\mathcal{A}_5 &\coloneqq \{(1, -2, 0), (0, 1, -2)\},\\
\mathcal{A}_6 &\coloneqq \{(1, -2, 0), (0, 2, -4), (-8, 0, 4)\},\\
\mathcal{A}_7 &\coloneqq \{(-1, 0, 4), 6I_b - 1\}.
\end{aligned}$$

   2.1 Repeat the subquestions of Exercise 1.



i	$\mathcal{A}_i$	asl	apl	asg	apg	$\mathcal{E}(\mathcal{A}_i)$	coh
5	$\{(1, -2, 0), (0, 1, -2)\}$	y	y	n	n	$\operatorname{posi}(\mathcal{A}_5 \cup \{I_a, I_b, I_c\})$	y
6	$\{(1, -2, 0), (0, 1, -2), (-2, 0, 1)\}$	n	n	n	n	$\mathcal{L}(\{a, b, c\})$	n
7	$\{(-\frac{1}{3}, 0, \frac{4}{3}), 2I_b - \frac{1}{3}\}$	y	y	n	n	$\operatorname{posi}(\mathcal{A}_7 \cup \{I_a\})$	y

   2.2 Represent $\mathcal{E}(\mathcal{A}_7)$ in the sum-one plane of $\mathcal{L}(\{a, b, c\})$.



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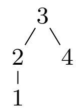
3. Repeat Exercise 1 for vector orderings defined by the cones

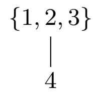
$$\mathcal{C}_1 \coloneqq \mathrm{posi}(\{(1, \tfrac{1}{10}), (0, 1)\})$$

i	$\mathcal{A}_i$	asl	apl	asg	apg	$\mathcal{E}(\mathcal{A}_i)$	coh
1	$\{(-1, \frac{1}{1000})\}$	y	y	n	n	$\mathrm{posi}(\{(-1, \frac{1}{1000}), (1, \frac{1}{10})\})$	y
2	$\{(-1, 0), (\frac{1}{2}, 1), (1, \frac{1}{2})\}$	y	y	n	n	$\mathrm{posi}(\{(-1, 0), (1, \frac{1}{10})\})$	y
3	$\{(-1, \frac{1}{1000}), (\frac{1}{2}, -1)\}$	n	n	n	n	$\mathcal{L}(\{a, b\})$	n
4	$\{(-\frac{1}{2}, 1), (1, -\frac{1}{2})\}$	y	y	y	y	$\mathrm{posi}(\mathcal{A}_4)$	y



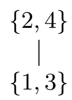
$$\mathcal{C}_2 \coloneqq \mathrm{posi}(\{(1, -\tfrac{1}{10}), (0, 1)\})$$

i	$\mathcal{A}_i$	asl	apl	asg	apg	$\mathcal{E}(\mathcal{A}_i)$	coh
1	$\{(-1, \frac{1}{1000})\}$	n	n	n	n	$\mathcal{L}(\{a, b\})$	n
2	$\{(-1, 0), (\frac{1}{2}, 1), (1, \frac{1}{2})\}$	n	n	n	n	$\mathcal{L}(\{a, b\})$	n
3	$\{(-1, \frac{1}{1000}), (\frac{1}{2}, -1)\}$	n	n	n	n	$\mathcal{L}(\{a, b\})$	n
4	$\{(-\frac{1}{2}, 1), (1, -\frac{1}{2})\}$	y	y	y	y	$\mathrm{posi}(\mathcal{A}_4)$	y



$$\mathcal{C}_3 \coloneqq \mathrm{posi}(\{(1, -\tfrac{1}{10}), (0, -1)\})$$

i	$\mathcal{A}_i$	asl	apl	asg	apg	$\mathcal{E}(\mathcal{A}_i)$	coh
1	$\{(-1, \frac{1}{1000})\}$	y	y	n	n	$\mathrm{posi}(\{(-1, \frac{1}{1000}), (1, -\frac{1}{10})\})$	y
2	$\{(-1, 0), (\frac{1}{2}, 1), (1, \frac{1}{2})\}$	n	n	n	n	$\mathcal{L}(\{a, b\})$	n
3	$\{(-1, \frac{1}{1000}), (\frac{1}{2}, -1)\}$	y	y	n	n	$\mathrm{posi}(\{(-1, \frac{1}{1000}), (1, -\frac{1}{10})\})$	y
4	$\{(-\frac{1}{2}, 1), (1, -\frac{1}{2})\}$	n	n	n	n	$\mathcal{L}(\{a, b\})$	n



4. Prove the Natural Extension Theorem.

   By construction, the natural extension $\mathcal{E}(\mathcal{A})$ must be included in any coherent extension, if they exist, as they must satisfy the constructive criteria and accepting partial gain: it is therefore the least committal one if it is coherent itself. This is the case if and only if it also avoids partial loss. From the definition of natural extension we see that $\mathcal{E}(\mathcal{A})$'s pointwise smallest gambles lie in $\mathrm{posi}(\mathcal{A})$ or $\mathcal{L}^+(\mathcal{X})$, which proves the necessary equivalence of $\mathcal{A}$ avoiding partial loss, i.e., $\mathrm{posi}(\mathcal{A}) \cap \mathcal{L}^-(\mathcal{X}) = \emptyset$, and $\mathcal{E}(\mathcal{A}) \cap \mathcal{L}^-(\mathcal{X}) = \emptyset$.

2 Derived coherent sets of desirable gambles & Combining sets of desirable gambles

1. Explicitly show that the transformation $\mathit{\Gamma}_\gamma$ associated to the surjective map

$$\gamma : \{0, 1\}^2 \to \{0, 1, 2\} : x \mapsto x_1 + x_2$$

   preserves coherence.

   The transformation $\mathit{\Gamma}_\gamma$ is defined by $\big(\mathit{\Gamma}_\gamma h\big)(x_1, x_2) = h(x_1 + x_2)$ for all gambles h on $\{0, 1, 2\}$ and (x_1, x_2) in $\{0, 1\}^2$. Linearity: $\big(\mathit{\Gamma}_\gamma(\lambda_1 h_1 + \lambda_2 h_2)\big)(x_1, x_2) = (\lambda_1 h_1 + \lambda_2 h_2)(x_1 + x_2) = \lambda_1 h_1(x_1 + x_2) + \lambda_2 h_2(x_1 + x_2)$. Monotonicity: $h > 0 \Rightarrow \mathit{\Gamma}_\gamma h > 0$ is immediate; $f \coloneqq \mathit{\Gamma}_\gamma h > 0 \Rightarrow h > 0$ follows from $h(0) = f(0, 0)$, $h(2) = f(1, 1)$, and $h(1) = f(0, 1) = f(1, 0)$.

   1.1 What slice of $\mathcal{L}(\{0, 1\}^2)$ does $\mathit{\Gamma}_\gamma$ generate? $\big(\mathit{\Gamma}_\gamma(\mathcal{L}(\{0, 1, 2\}))\big) = \big\{f \in \mathcal{L}(\{0, 1\}^2) \colon f(0, 1) = f(1, 0)\big\}$

   1.2 What is the partition associated to γ? $\mathcal{B}_\gamma = \big\{\{(0, 0)\}, \{(0, 1), (1, 0)\}, \{(1, 1)\}\big\}$

2. Show that the transformation $\mathit{\Gamma} : \mathcal{L}(\{0, 1, 2\}) \to \mathcal{L}([0, 1])$ that maps a gamble g to the parabola

$$g(0)(1 - \theta)^2 + g(1)\theta(1 - \theta) + 2g(2)\theta^2$$

   in θ does not preserve coherence, by considering $1 - 4\theta + 4\theta^2$.

   If $(\mathit{\Gamma} g)(\theta) = 1 - 4\theta + 4\theta^2 > 0$ for $0 \leq \theta \leq 1$, then after solving a linear system, we find $g = \big(g(0), g(1), g(2)\big) = (1, -2, 1) \not> 0$.

   2.1 Describe the linear subspace of $\mathcal{L}([0, 1])$ generated by $\mathit{\Gamma}$. All parabola on $[0, 1]$: $\mathit{\Gamma}\big(\mathcal{L}(\{0, 1, 2\})\big)$.

   2.2 Define a vector ordering on this subspace that makes $\mathit{\Gamma}$ preserve coherence.

   Use the cone $\mathcal{C} \coloneqq \mathit{\Gamma}\big(\mathcal{L}^+(\{0, 1, 2\})\big)$.

3. Take $\mathcal{E}(\mathcal{A}_7)$ from Exercise 2.2 of the previous series. Let $\mathcal{D} \coloneqq \mathcal{E}(\mathcal{A}_7)$.

   3.1 Calculate its conditionals for all nonempty events of $\{a, b, c\}$, give the extreme-ray representation in all three formats.

$$\begin{aligned}
\mathcal{D}|\{a,b,c\} &= \mathcal{D} \sim \mathcal{D} \sim \mathcal{D},\\
\forall x \in \{a,b,c\} : \mathcal{D}|\{x\} &= \mathbb{R}^+ \sim \operatorname{posi}(\{I_x\}) \sim \{I_x\} \cup \textstyle\bigcup_{y\in\{a,b,c\}\setminus\{x\}}\{I_y, -I_y\},\\
\mathcal{D}|\{a,b\} &= \operatorname{posi}\big(\{I_a, (-\tfrac{1}{4}, 1)\}\big) \sim \operatorname{posi}\big(\{I_a, (-\tfrac{1}{4}, 1, 0)\}\big) \sim \operatorname{posi}\big(\{I_a, (-\tfrac{1}{4}, 1, 0), I_c, -I_c\}\big),\\
\mathcal{D}|\{b,c\} &= \operatorname{posi}\big(\{I_c, (1, -\tfrac{1}{5})\}\big) \sim \operatorname{posi}\big(\{I_c, (0, 1, -\tfrac{1}{5})\}\big) \sim \operatorname{posi}\big(\{I_c, (0, 1, -\tfrac{1}{5}), I_a, -I_a\}\big),\\
\mathcal{D}|\{a,c\} &= \operatorname{posi}\big(\{I_a, (-\tfrac{1}{4}, 1)\}\big) \sim \operatorname{posi}\big(\{I_a, (-\tfrac{1}{4}, 0, 1)\}\big) \sim \operatorname{posi}\big(\{I_a, (-\tfrac{1}{4}, 0, 1), I_b, -I_b\}\big).
\end{aligned}$$

   3.2 Calculate its marginals for all partitions of $\{a, b, c\}$.

$$\mathcal{D}_{\{\{a\},\{b,c\}\}} = \operatorname{posi}\big(\{(-1,0), (-\tfrac{1}{2}, 1)\}\big), \qquad \mathcal{D}_{\{\{b\},\{a,c\}\}} = \operatorname{posi}\big(\{I_{\{a,c\}}, (1, -\tfrac{1}{5})\}\big),$$
$$\mathcal{D}_{\{\{c\},\{a,b\}\}} = \operatorname{posi}\big(I_c, (-\tfrac{1}{5}, 1)\big), \qquad \mathcal{D}_{\{\{a\},\{b\},\{c\}\}} = \mathcal{D}, \quad \mathcal{D}_{\{\{a,b,c\}\}} = \mathbb{R}^+.$$

   3.3 Calculate the marginal extensions of the appropriate derived conditionals and marginals for all partitions of $\{a, b, c\}$.

$$\begin{aligned}
\mathcal{E}\big(\varGamma_{\{\{a,b,c\}\}}\mathcal{D}_{\{\{a,b,c\}\}} \cup \dagger_{\emptyset}\mathcal{D}|\{a,b,c\}\big) &= \mathcal{D},\\
\mathcal{E}\big(\varGamma_{\{\{a\},\{b\},\{c\}\}}\mathcal{D}_{\{\{a\},\{b\},\{c\}\}} \cup \textstyle\bigcup_{x\in\{a,b,c\}}\dagger_{\{a,b,c\}\setminus\{x\}}\mathcal{D}|\{x\}\big) &= \mathcal{D},\\
\mathcal{E}\big(\varGamma_{\{\{a\},\{b,c\}\}}\mathcal{D}_{\{\{a\},\{b,c\}\}} \cup \dagger_{\{b,c\}}\mathcal{D}|\{a\} \cup \dagger_{\{a\}}\mathcal{D}|\{b,c\}\big) &= \operatorname{posi}\big(\{I_a, I_c, (-\tfrac{1}{3}, \tfrac{2}{3}, \tfrac{2}{3}), (0, \tfrac{5}{4}, -\tfrac{1}{4})\}\big),\\
\mathcal{E}\big(\varGamma_{\{\{b\},\{a,c\}\}}\mathcal{D}_{\{\{b\},\{a,c\}\}} \cup \dagger_{\{a,c\}}\mathcal{D}|\{b\} \cup \dagger_{\{b\}}\mathcal{D}|\{a,c\}\big) &= \mathcal{D},\\
\mathcal{E}\big(\varGamma_{\{\{c\},\{a,b\}\}}\mathcal{D}_{\{\{c\},\{a,b\}\}} \cup \dagger_{\{a,b\}}\mathcal{D}|\{c\} \cup \dagger_{\{c\}}\mathcal{D}|\{a,b\}\big) &= \operatorname{posi}\big(\{I_a, (-\tfrac{1}{3}, \tfrac{4}{3}, 0), (\tfrac{5}{9}, \tfrac{5}{9}, -\tfrac{1}{9}), (-\tfrac{1}{3}, 0, \tfrac{4}{3})\}\big).
\end{aligned}$$

4. Prove the Marginal Extension Theorem.

   We need to prove that $\mathcal{A}$ avoids partial loss. Assume it does not, then there will be a finite subset $\mathcal{B}'$ of $\mathcal{B}$, a gamble $h \in \mathcal{D}_{\mathcal{B}}$, and gambles $g_b \in \mathcal{D}|B$, $B \in \mathcal{B}'$, such that $\varGamma_{\mathcal{B}}h + \sum_{B\in\mathcal{B}'}\dagger_{B^c}g_B = 0$. Coming from the $\mathcal{B}$-marginal $\mathcal{D}_{\mathcal{B}}$, $\varGamma_{\mathcal{B}}h$ is constant on elements of $\mathcal{B}$; because $\mathcal{D}_{\mathcal{B}}$ is coherent, it will be positive on at least one event $C \in \mathcal{B}'$. Being contingent gambles, the $\dagger_{B^c}g_B$, $B \in \mathcal{B}'$, have disjunct support. This means $g_C < 0$, contradicting separate coherence.

3 Partial preference orders

1. Possibility space $\{a, b\}$.

   1.1 Which of $(-4, 3)$, $(-3, 4)$, and $(3, -3)$ belong to $\mathcal{D}_{\succ}$, $\mathcal{D}_{\succcurlyeq}$, both, or neither, when $(5, -2) \approx (-2, 5)$.

   This implies $\{(1, -1), (-1, 1)\} \subset \mathcal{D}_{\succcurlyeq}$, so $(-4, 3) \notin \mathcal{D}_{\succcurlyeq}$, $(-3, 4) \in \mathcal{D}_{\succ}$, and $(3, -3) \in \mathcal{D}_{\succcurlyeq} \setminus \mathcal{D}_{\succ}$.

   1.2 Which, or both, or neither of $\{(-1, 1)\}$ and $\{(2, -3)\}$ is compatible with $(5, -3) \asymp (4, -1)$.

   This implies $\mathcal{D}_{\succ} \subset \operatorname{posi}\{(\tfrac{1}{2}, -1), (-\tfrac{1}{2}, 1), (1, 1)\}$, so $\{(2, -3)\}$ is compatible, but $\{(-1, 1)\}$ is not.

2. Prove the equivalence of the rationality criteria for strict preference and strict desirability.

   Transitivity, in terms of desirability, becomes $f - g \in \mathcal{D} \wedge g - h \in \mathcal{D} \Rightarrow f - h \in \mathcal{D}$, which is equivalent to the addition criterion. Mixture-independence, in terms of desirability, becomes $f - g \in \mathcal{D} \Leftrightarrow \mu(f - g) \in \mathcal{D}$ for $0 < \mu \leq 1$, which, together with the addition criterion, is equivalent to the positive scaling criterion. Monotonicity, in terms of desirability, becomes $f - g \in \mathcal{L}^+(\mathcal{X}) \Rightarrow f - g \in \mathcal{D}$, accepting partial gain. Irreflexivity corresponds to $0 \notin \mathcal{D}$; combining this with monotonicity, we get $g - f \in \mathcal{L}_0^-(\mathcal{X}) \Rightarrow g - f \notin \mathcal{D}$, avoiding nonpositivity.

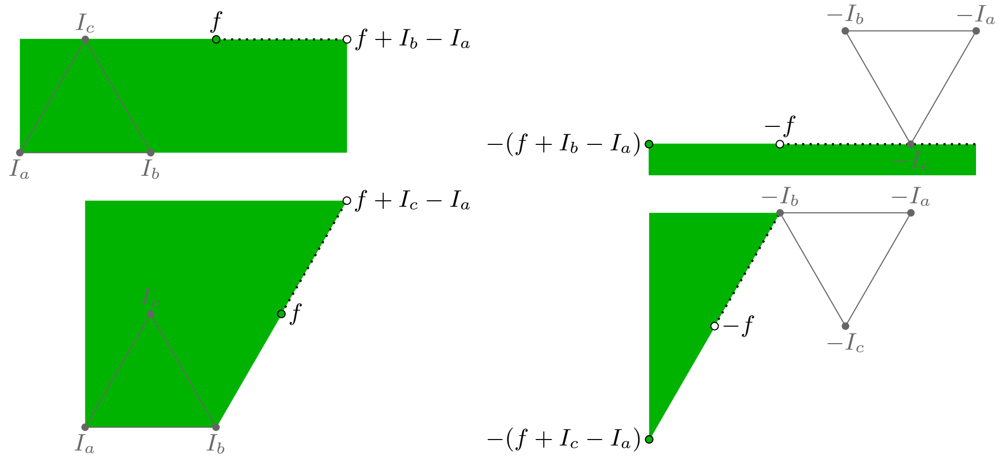
3. Prove that $\succcurlyeq$ satisfies the rationality criteria of nonstrict preference (assume they are equivalent to those for nonstrict desirability).

   Reflexivity is immediate, because $(f - f) + \mathcal{D}_{\succ} = \mathcal{D}_{\succ}$. Given $(g - h) + \mathcal{D}_{\succ} \subseteq \mathcal{D}_{\succ}$ and $(f - g) + \mathcal{D}_{\succ} \subseteq \mathcal{D}_{\succ}$, additivity implies $(f - h) + \mathcal{D}_{\succ} \subseteq \mathcal{D}_{\succ}$, so transitivity holds. To prove that mix-independence holds, i.e., $(f - g) + \mathcal{D}_{\succ} \subseteq \mathcal{D}_{\succ} \Leftrightarrow \mu(f - g) + \mathcal{D}_{\succ} \subseteq \mathcal{D}_{\succ}$, realize that positive scaling implies $\mathcal{D}_{\succ} = \mu\mathcal{D}_{\succ}$; then the equivalence's right-hand side can be written as $\mu(f - g) + \mu\mathcal{D}_{\succ} \subseteq \mu\mathcal{D}_{\succ}$, so that we see both sides only formally differ, in their arbitrary scaling factor. Monotonicity's first right-hand conjunct follows from $f - g \in \mathcal{L}^+(\mathcal{X}) \subseteq \mathcal{D}_{\succ}$ and additivity; the second, $(g - f) + \mathcal{D}_{\succ} \nsubseteq \mathcal{D}_{\succ}$, follows from the fact that $\tfrac{1}{2}(f - g) \in \mathcal{D}_{\succ}$, but $\tfrac{1}{2}(g - f) \notin \mathcal{D}_{\succ}$.

4 Maximally committal sets of strictly desirable gambles

1. Possibility space $\{a, b, c\}$; let $f := (-1, 1, 1)$ be an extreme ray of a maximal set of desirable gambles.
   1.1 Draw the intersection with the sum-one plane of the ones for which respectively $f + I_b - I_a$ and $f + I_c - I_a$ are nonstrictly desirable.
   1.2 Also draw their intersection with the sum-minus one plane.

   Note that, because maximal sets of desirable gambles are halfspaces, we cannot draw their whole intersection; we give subsets that make the essential features apparent.



2. Prove the Characterization of Maximal Sets of Desirable Gambles

   Coherence, avoiding nonpositivity to be precise, makes $f \in \mathcal{D} \Rightarrow -f \notin \mathcal{D}$ a necessity for all nonzero gambles f on $\mathcal{X}$. So we have to prove that maximality of $\mathcal{D}$ is equivalent to $-f \notin \mathcal{D} \Rightarrow f \in \mathcal{D}$ for all nonzero gambles f. First necessity; assume $\mathcal{D}$ is maximal but nevertheless $\{f, -f\} \cap \mathcal{D} = \emptyset$ for some nonzero gamble f. Then both $\{f\} \cup \mathcal{D}$ and $\{-f\} \cup \mathcal{D}$ incur nonpositivity, which, because $\mathcal{D}$ is coherent and thus a cone excluding the zero gamble, means both $0 \in \operatorname{posi}(\{f\}) + \mathcal{D}$ and $0 \in \operatorname{posi}(\{-f\}) + \mathcal{D}$. Or, in other words, $\{f, -f\} \subset \mathcal{D}$, a contradiction.

   Now sufficiency; if $|\{f, -f\} \cap \mathcal{D}| = 1$ for all nonzero gambles f, then any set strictly encompassing $\mathcal{D}$ incurs nonpositivity.

3. Prove the Maximal Sets and Natural Extension Corollary

   That the least committal extension is a subset follows from avoiding nonpositivity. Assume it is a strict subset, then $\mathcal{A}' := \bigcap \hat{\mathbb{D}}_{\mathcal{A}} \setminus \mathcal{E}(\mathcal{A}) \neq \emptyset$. So $\mathcal{A} \cup -\mathcal{A}'$ avoids nonpositivity and thus $\mathcal{E}(\mathcal{A} \cup -\mathcal{A}')$ is a coherent extension of $\mathcal{A}$, but it is not included in $\bigcap \hat{\mathbb{D}}_{\mathcal{A}}$, a contradiction.

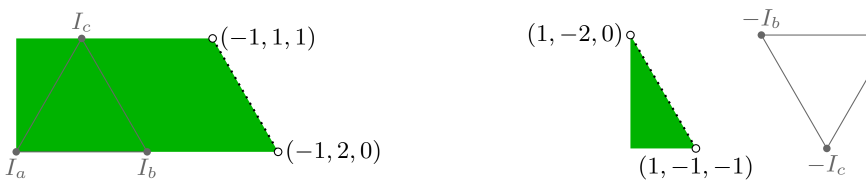
4. Prove the Maximality Preserving Transformations Proposition

   Let $\mathcal{D} \in \hat{\mathbb{D}}(\mathcal{X})$ and let $\Gamma : \mathcal{L}(\mathcal{Z}) \to \mathcal{L}(\mathcal{X})$ be a coherence preserving transformation. Assume that the derived set of desirable gambles $\mathcal{D}_\Gamma$ is nonmaximal, i.e., that there is some gamble h on $\mathcal{Z}$ such that $\{h, -h\} \cap \mathcal{D}_\Gamma = \emptyset$. Then, because of Γ's self-conjugacy, $\{\Gamma h, -\Gamma h\} \cap \mathcal{D} = \emptyset$, a contradiction.

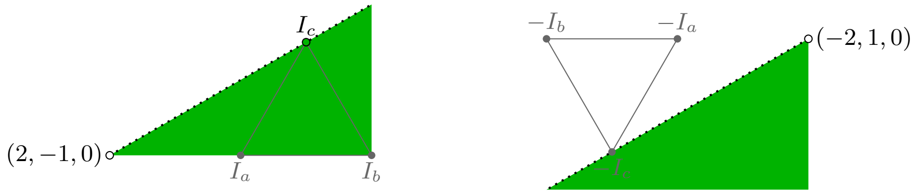
5 Relationships with other, nonequivalent models

1. Possibility space $\{a, b, c\}$; draw the intersection of $\mathcal{D}_{P_i}$ with the sum-one and sum-minus one planes for the linear previsions defined by

$$P_1(f) = \tfrac{1}{2} f(a) + \tfrac{1}{4} f(b) + \tfrac{1}{4} f(c)$$

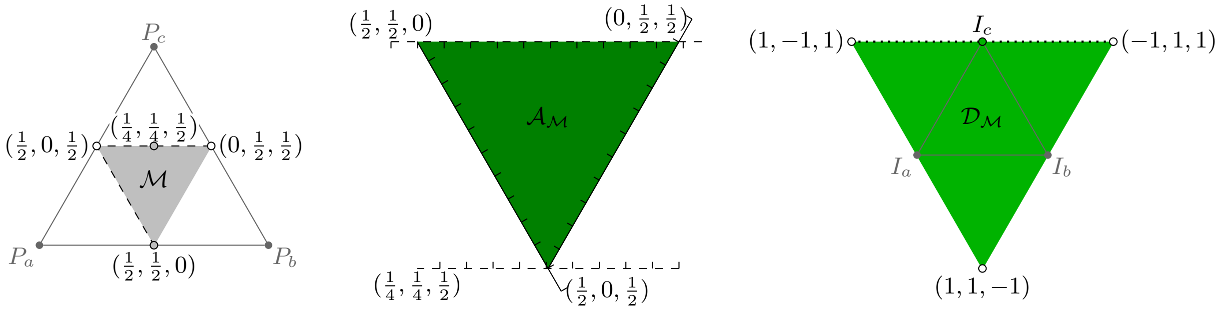


$$P_2(f) = \frac{1}{3}f(a) + \frac{2}{3}f(b)$$

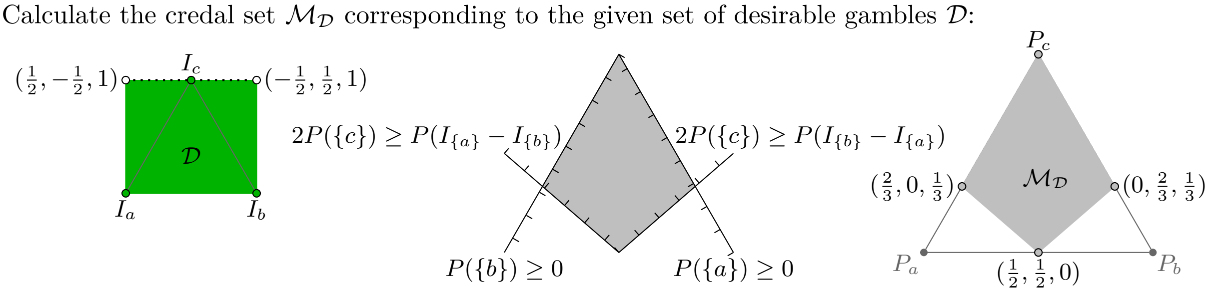


Note that, because maximal sets of desirable gambles are halfspaces, we cannot draw their whole intersection; we give subsets that make the essential features apparent.

2. Calculate the set of desirable gambles $\mathcal{D}_{\mathcal{M}}$ corresponding to the given credal set $\mathcal{M}$:



3. Calculate the credal set $\mathcal{M}_{\mathcal{D}}$ corresponding to the given set of desirable gambles $\mathcal{D}$:



4. Give the corresponding simplified variants for all the sets of desirable gambles appearing up until now in this exercise series.

   To be done introspectively.

5. Possibility space $\{a,b,c\}$; a lower prevision $\underline{P}$ is specified as follows: the lower probability of $\{c\}$ and $\{b,c\}$ are, respectively, $\frac{1}{6}$ and $\frac{1}{4}$; the supremum upper buying price for $(-3,3,-2)$ is -2.

   5.1 Calculate $\mathcal{D}_{\underline{P}}$ and use it to check …

   The given assessments imply $(-\frac{1}{3},-\frac{1}{3},\frac{5}{3})$, $(-\frac{1}{5},\frac{3}{5},\frac{3}{5})$, and $(-\frac{1}{4},\frac{5}{4},0)$ are marginally desirable; $\mathcal{D}_{\underline{P}}$ is the natural extension of their sum with $\mathbb{R}^+$.

   5.2 whether $\underline{P}$ avoids sure loss,

   Yes, easy to check graphically: $\mathcal{D}_{\underline{P}}$ is not the whole space.

   5.3 whether $\underline{P}$ is coherent,

   No, easy to check graphically: $(-\frac{1}{5},\frac{3}{5},\frac{3}{5})$ lies strictly inside $\mathcal{D}_{\underline{P}}$.

   5.4 calculate the natural extension of $\underline{P}$ to $I_{\{a,b\}}$, $I_{\{b,c\}}$, and $I_{\{c,a\}}$.

   We know that the natural extension of an arbitrary gamble f is defined by marginally desirability of $\beta(f - \underline{E}(f))$, for any $\beta \geq 0$. Take f to be $I_{\{a,b\}}$, $I_{\{b,c\}}$, and $I_{\{c,a\}}$, respectively, then this gamble is respectively constant on $\{a,b\}$, $\{b,c\}$, and $\{c,a\}$. Using the graphical representation in the sum-one plane, one can find the intersection of the lines corresponding to these partially constant gambles with the marginally desirable gambles. We find $\underline{E}(\{a,b\}) = 0$, $\underline{E}(\{b,c\}) = \frac{11}{37}$, and $\underline{E}(\{a,c\}) = \frac{1}{6}$.